DYNAMIC HEAT TRANSFER IN BUILDINGS: SELECTING A REDUCED-ORDER MODEL

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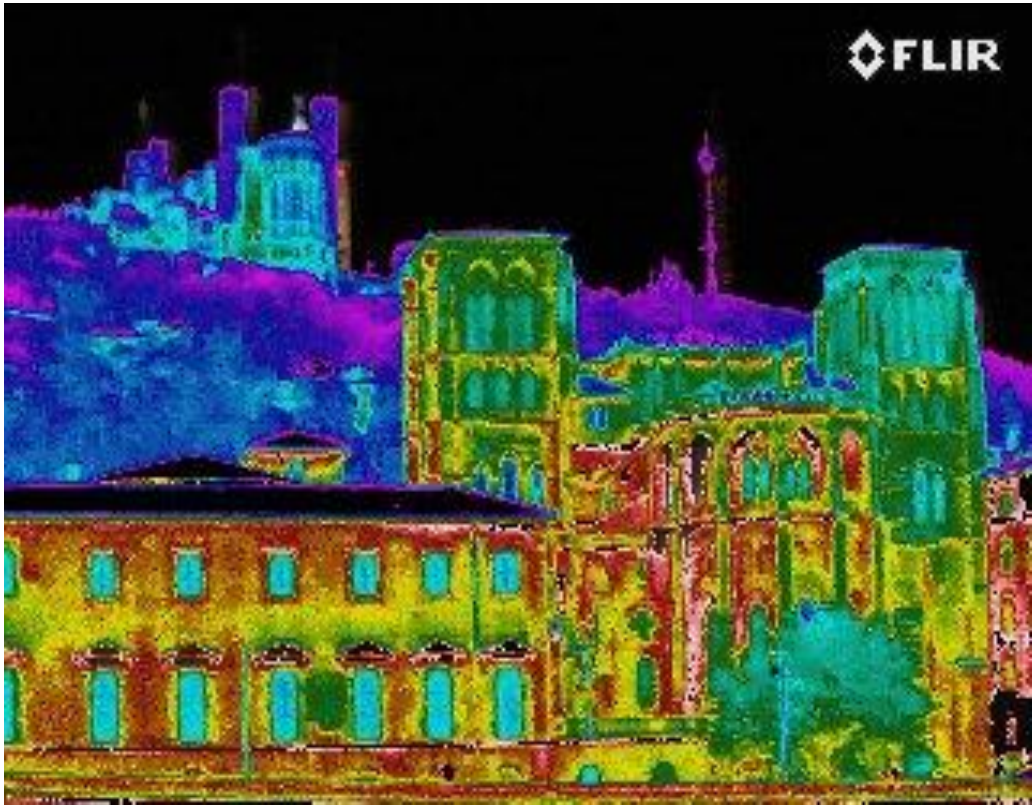
INTRODUCTION

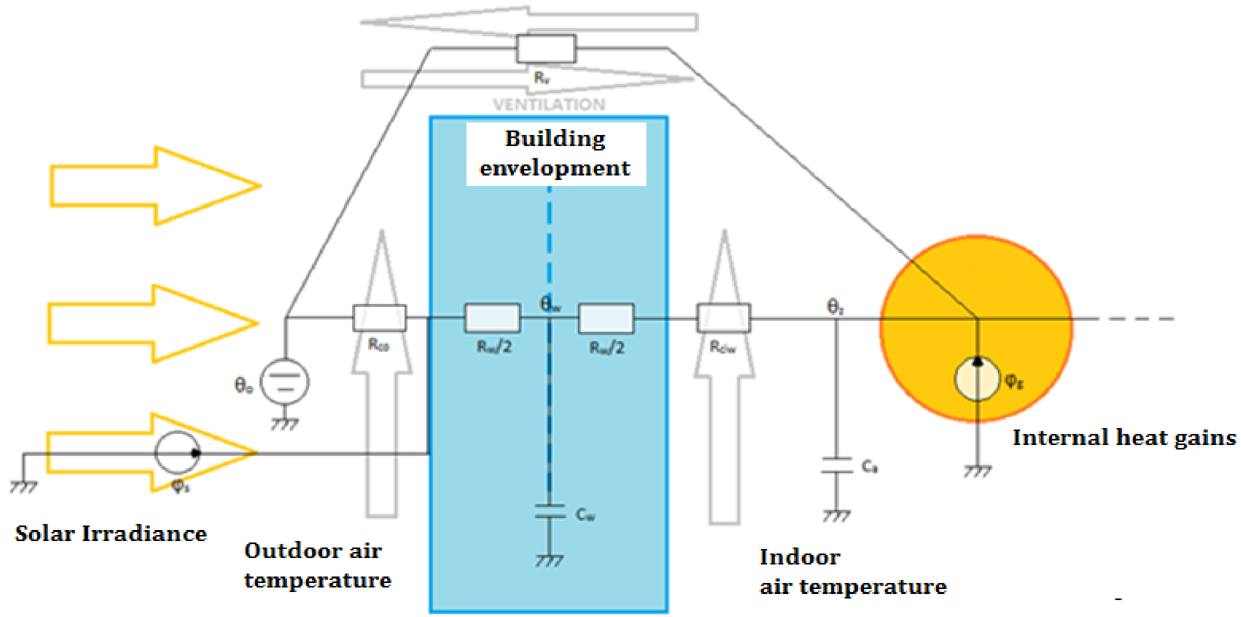
Buildings are **energy** systems and there is a necessity to quantify heat **exchange** to their **surroundings**



Lyon (France)

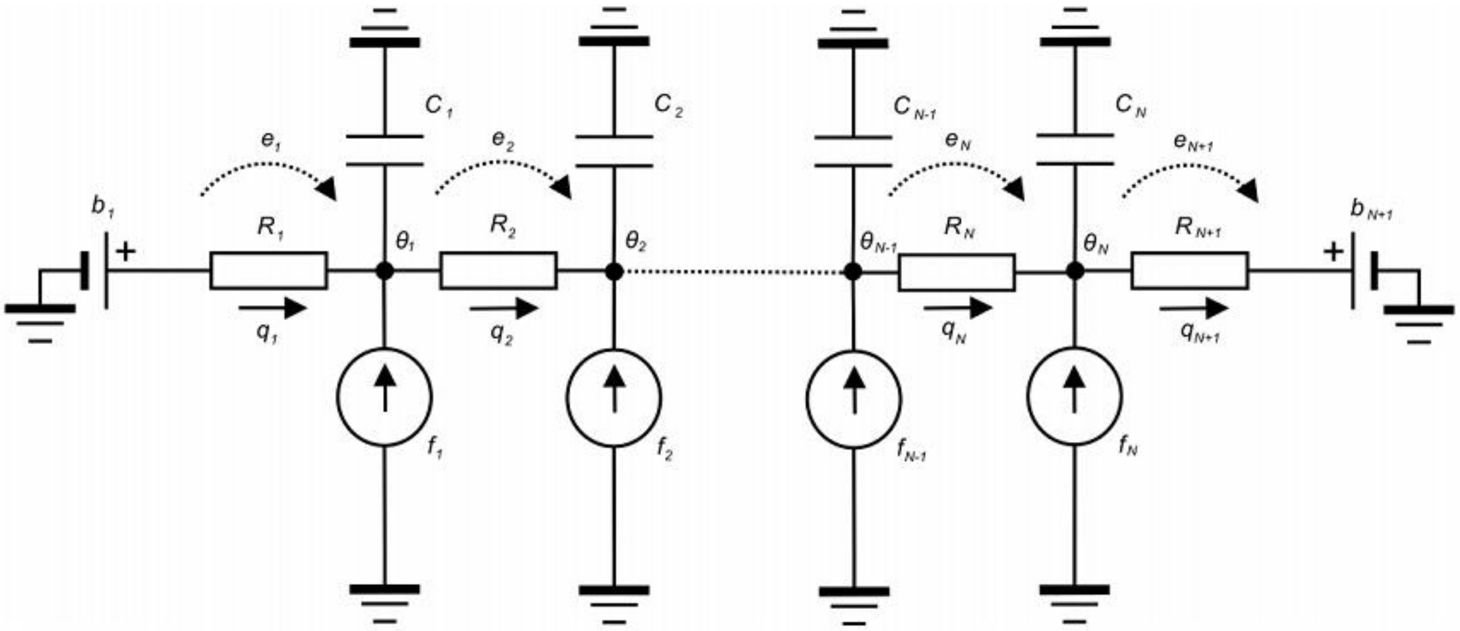
INTRODUCTION

Modelling heat transfer in buildings



PROBLEM: what is the **optimal order of the model** to be used in simulation and experimental parameter identification **for building energy efficiency estimation?**

THERMAL NETWORK



LEGEND

- C_l thermal capacity in node l, JK^{-1}
- e_k temperature difference over the thermal resistance on branch k, K
- b_k temperature source on branch k, K
- R_k thermal resistance on branch k, KW^{-1}
- θ_l temperature of node l, K
- q_k heat transfer rate on branch k, W
- f_l heat rate source in node l, W

HEAT EQUATION

$$\rho c \frac{\partial \theta}{\partial t} = -\nabla \cdot (-\kappa \, \nabla \theta) + p$$

DIFFERENTIAL AND ALGEBRAIC EQUATIONS

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T\mathbf{G}\mathbf{A}\boldsymbol{\theta} + \mathbf{A}^T\mathbf{G}\mathbf{b} + \mathbf{f}$$

NOMENCLATURE

- θ is the function of temperature distribution in the medium, K,
- ρ - the medium density, kg m^{-3},
- c - the medium heat capacity, J kg^{-1} K^{-1},
- ∇ - the gradient operator, m^{-1},
- $\nabla \cdot$ - the divergence operator, m^{-1},
- κ - the medium thermal conductivity, W m^{-1} K^{-1}, and
- p - the function of heat rate sources supplied to the solid, W m^{-3}.

- $\boldsymbol{\theta}$ is the vector of temperatures in the nodes, K,
- $\mathbf{C}$ - the diagonal matrix of thermal capacities, JK^{-1},
- $\mathbf{A}$ - the incidence matrix of the thermal network,
- $\mathbf{A}^T$ - the transpose of the incidence matrix,
- $\mathbf{G}$ - the diagonal matrix of thermal conductivities, WK^{-1},
- $\mathbf{b}$ - the vector of temperatures sources on the branches, K, and
- $\mathbf{f}$ - the vector of heat rate sources, W.

METHODOLOGY

- **Thermal models** for the study of heat transfer in **buildings**
- From **heat equation** to a set of **DAE**
- From DAE to **state-space**
- From state-space to **transfer function matrix**

From DAE to State-Space

Partial differential heat equation

$$\rho c \frac{\partial \theta}{\partial t} = \nabla \cdot (\kappa \nabla \theta) + p$$

Set of differential algebraic equations (DAE)

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^{\mathrm{T}}\mathbf{G}\mathbf{A}\boldsymbol{\theta} + -\mathbf{A}^{\mathrm{T}}\mathbf{G}\mathbf{b} + \mathbf{f}$$

State-state representation:

$$\begin{aligned} \dot{\boldsymbol{\theta}}_C &= \mathbf{A}_S\boldsymbol{\theta} + \mathbf{B}_S\mathbf{u} \\ \boldsymbol{\theta}_0 &= \mathbf{C}_S\boldsymbol{\theta} + \mathbf{D}_S\mathbf{u} \end{aligned}$$

From State-Space to Transfer Function

State-space: Inputs($\mathbf{u}$)/State variables($\boldsymbol{\theta}_C$)/Outputs($\boldsymbol{\theta}_0$) relation

$$\dot{\boldsymbol{\theta}}_C = \mathbf{A}_S\boldsymbol{\theta}_C + \mathbf{B}_S\mathbf{u}$$
$$\boldsymbol{\theta}_0 = \mathbf{C}_S\boldsymbol{\theta}_C + \mathbf{D}_S\mathbf{u}$$

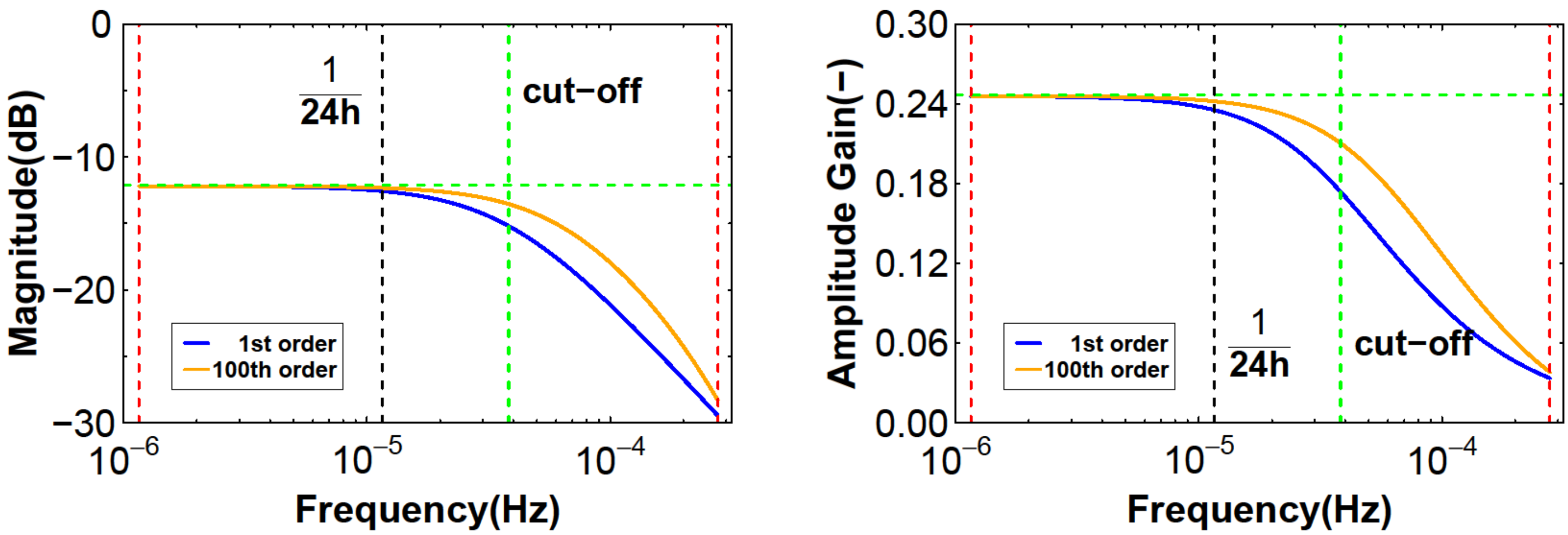
Transfer function matrix: Inputs($\mathbf{u}$)/Outputs($\boldsymbol{\theta}_0$) relation

$$\boldsymbol{\theta}_0 = (\mathbf{C}_S(s\mathbf{I} - \mathbf{A}_S)^{-1}\mathbf{B}_S + \mathbf{D}_S)\mathbf{u}$$

$$\mathbf{H}_S = \mathbf{C}_S(s\mathbf{I} - \mathbf{A}_S)^{-1}\mathbf{B}_S + \mathbf{D}_S$$

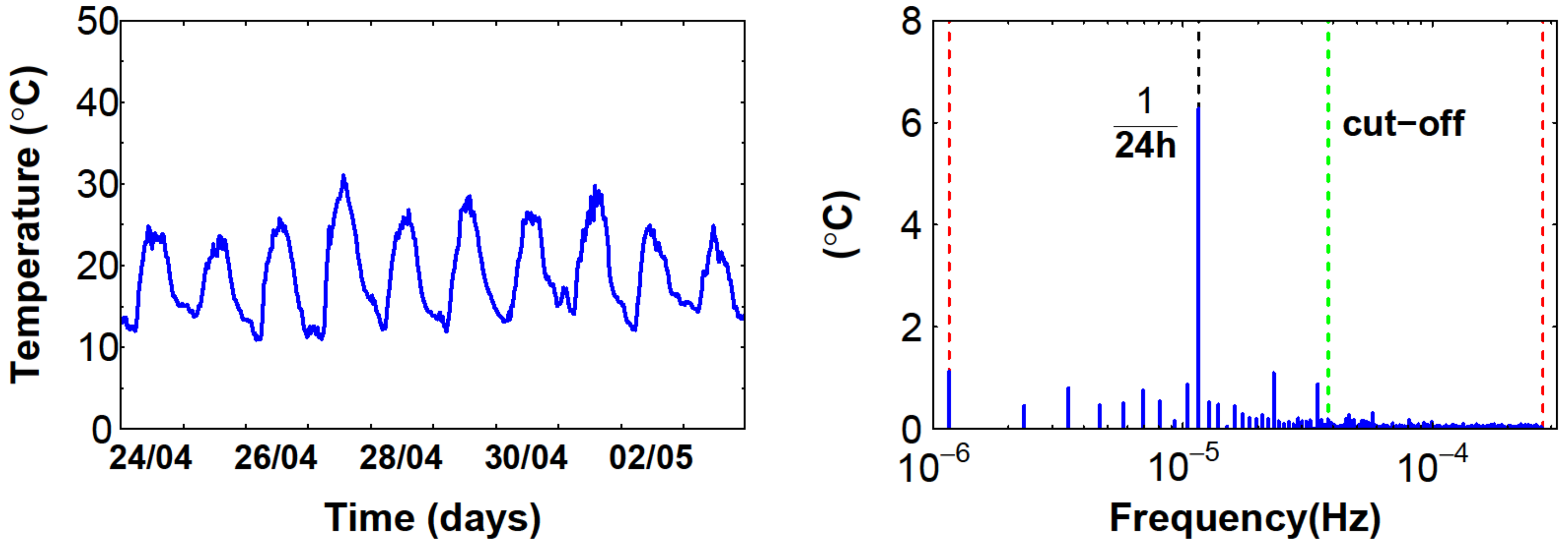
FREQUENCY STUDY

Each **transfer function** is given by the system characteristics for a particular output regarding to each particular input



FREQUENCY STUDY

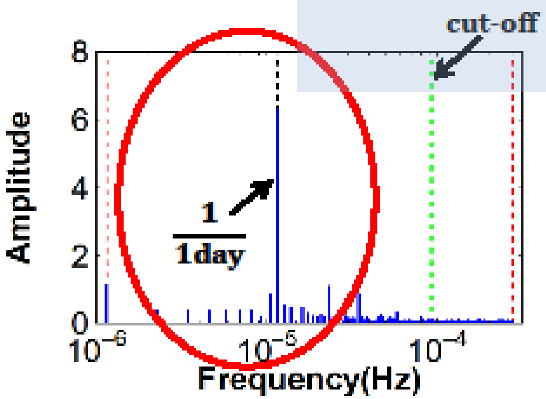
The frequency **spectrum** of the measured **inputs** (outdoor temperature) can be obtained using the fast Fourier transform



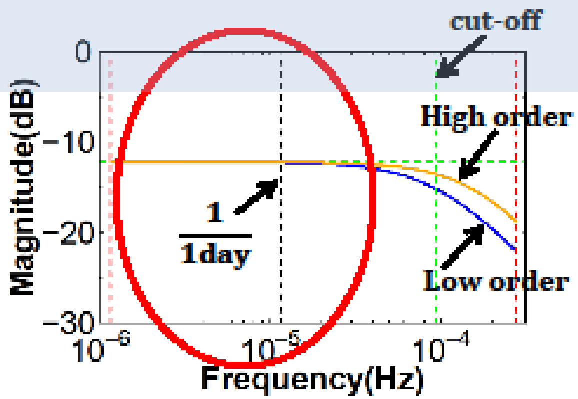
Criterion for model order selection

Input signal presents different amplitudes depending on frequency → The process filters the input depending on frequency → Output signal is measurable when input amplitudes are large enough within a frequency domain

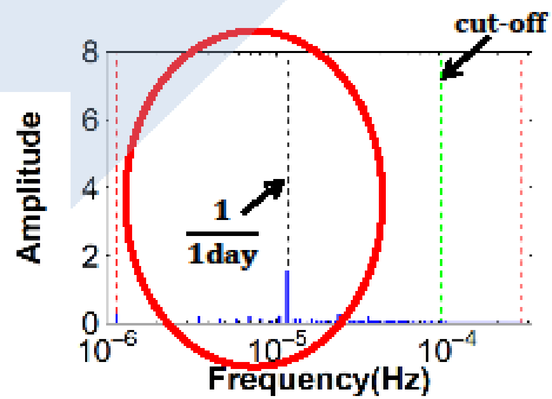
INPUT (ω) * PROCESS(ω) = OUTPUT(ω)



Frequency spectrum of the input



Bode amplitude plot of the system



Frequency spectrum of the output

CONCLUSIONS

- There is a **need** to **estimate building energy efficiency** using dynamic thermal models.
- From **heat equation** the dynamic model is of **infinite order** and it needs to be reduced for practical purposes.
- The **order** of the model can be **chosen** doing a **frequency analysis** of measurements combined to the transfer function.
- The **differences** between a **low** and a **high** model **order** are **negligible** in a wide range of frequencies.
- Then, the use of **reduced order model** is **justified** and **facilitates** energy efficiency **estimation** in buildings.

Thanks for your attention

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