

DYNAMIC HEAT TRANSFER IN WALLS: HEAT FLUX METERS LIMITATION

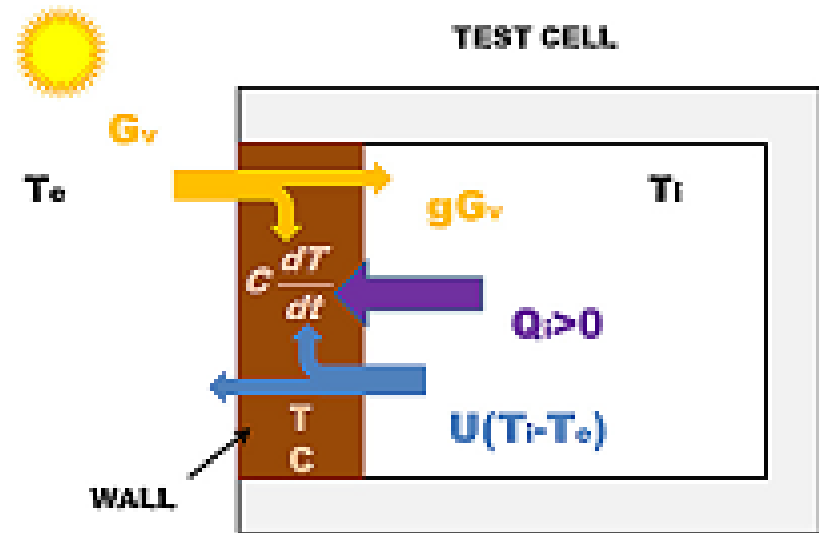
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INTRODUCTION

Heat transfer through walls needs to be characterized.
Test cells are usually used for this purpose.



Naveros et al. 2013 [9]

INTRODUCTION

Inputs and outputs are measured for the thermal characterization of the wall

Outdoor temperature



Indoor temperature



Solar irradiance



Heat flux density



Naveros et al. 2013 [9]

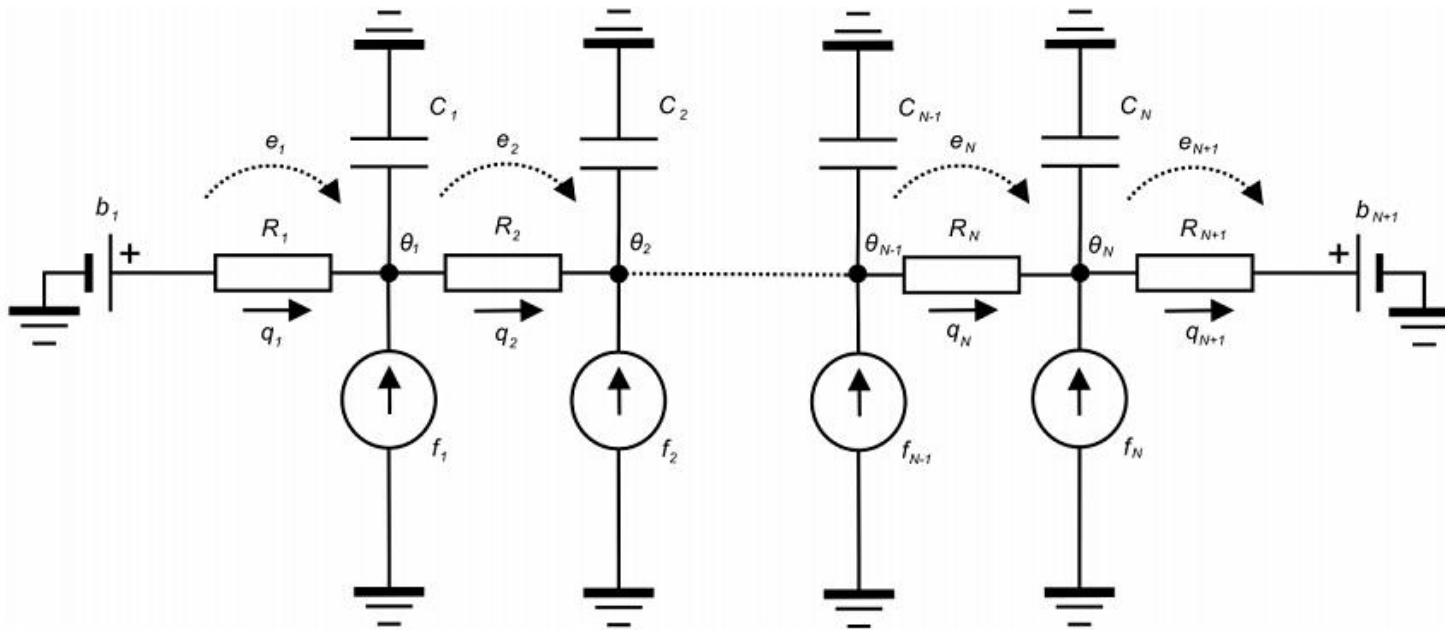
METHODOLOGY

- **Thermal models** for the study of heat transfer in walls
- From **heat equation** to **transfer function matrix**
- Each **transfer function** is given by the system characteristics for a **particular output** regarding to each **particular input**

THERMAL NETWORK

LEGEND

- C_l thermal capacity in node l , JK^{-1}
 e_k temperature difference over the thermal resistance on branch k , K
 b_k temperature source on branch k , K
 R_k thermal resistance on branch k , KW^{-1}
 θ_l temperature of node l , K
 q_k heat transfer rate on branch k , W
 f_l heat rate source in node l , W



HEAT EQUATION

$$\rho c \frac{\partial \theta}{\partial t} = -\nabla \cdot (-\kappa \nabla \theta) + p$$

DIFFERENTIAL AND ALGEBRAIC EQUATIONS

$$\mathbf{C}\dot{\boldsymbol{\theta}} = -\mathbf{A}^T \mathbf{G} \mathbf{A} \boldsymbol{\theta} + \mathbf{A}^T \mathbf{G} \mathbf{b} + \mathbf{f}$$

NOMENCLATURE

- θ is the function of temperature distribution in the medium, K ,
 ρ - the medium density, kg m^{-3} ,
 c - the medium heat capacity, $\text{J kg}^{-1} \text{K}^{-1}$,
 ∇ - the gradient operator, m^{-1} ,
 $\nabla \cdot$ - the divergence operator, m^{-1} ,
 κ - the medium thermal conductivity, $\text{W m}^{-1} \text{K}^{-1}$, and
 p - the function of heat rate sources supplied to the solid, W m^{-3} .

- $\boldsymbol{\theta}$ is the vector of temperatures in the nodes, K ,
 \mathbf{C} - the diagonal matrix of thermal capacities, JK^{-1} ,
 \mathbf{A} - the incidence matrix of the thermal network,
 \mathbf{A}^T - the transpose of the incidence matrix,
 \mathbf{G} - the diagonal matrix of thermal conductivities, WK^{-1} ,
 \mathbf{b} - the vector of temperatures sources on the branches, K , and
 \mathbf{f} - the vector of heat rate sources, W .

Transfer Matrix

State-state: Inputs(**u**)/State variables(**x**)/Outputs(**y**) relation

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}_S \mathbf{x} + \mathbf{B}_S \mathbf{u} \\ \mathbf{y} &= \mathbf{C}_S \mathbf{x} + \mathbf{D}_S \mathbf{u}\end{aligned}$$

Transfer function matrix:

$$\mathbf{H}_S = \mathbf{C}_S (s\mathbf{I} - \mathbf{A}_S)^{-1} \mathbf{B}_S + \mathbf{D}_S$$

$$\mathbf{H}_S = [\mathbf{H}_1 \quad \mathbf{H}_2]^T = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \end{bmatrix}$$

HEAT FLUX AS OUTPUT

If the **heat flux** density is chosen as **output**, the observation equation is:

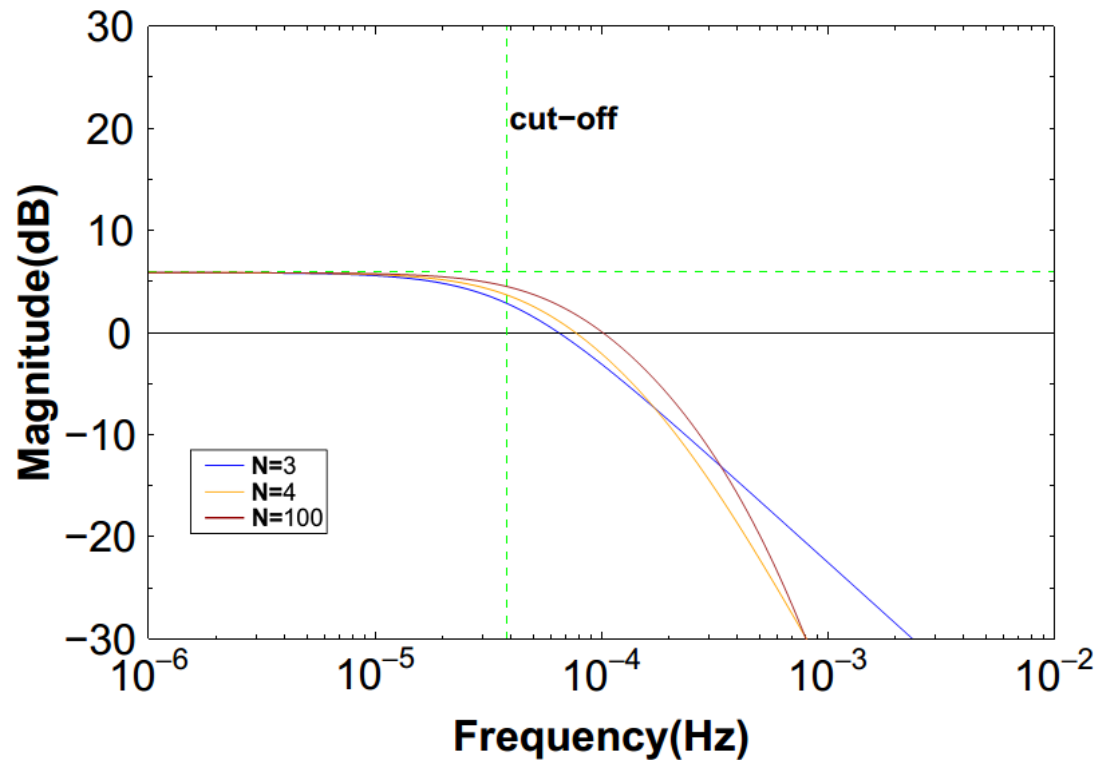
$$Q_i = \frac{1}{S} \frac{R_3^{-1} R_{si}^{-1}}{R_3^{-1} + R_{si}^{-1}} (T_i - \theta)$$

The **transfer function** will **filter** the different **inputs** as a function of the **frequency** to give the heat flux density output.

Strictly Proper Transfer Function

Input: Outdoor temperature

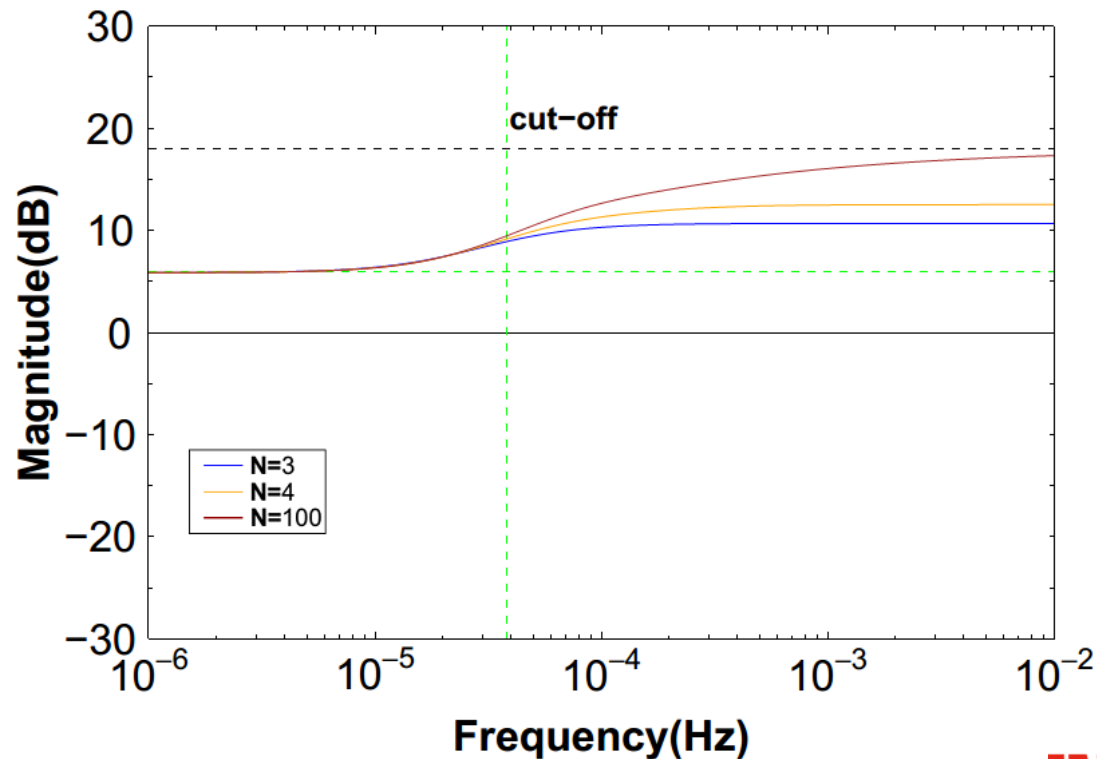
Output: Inside heat flux density



Proper Transfer Function

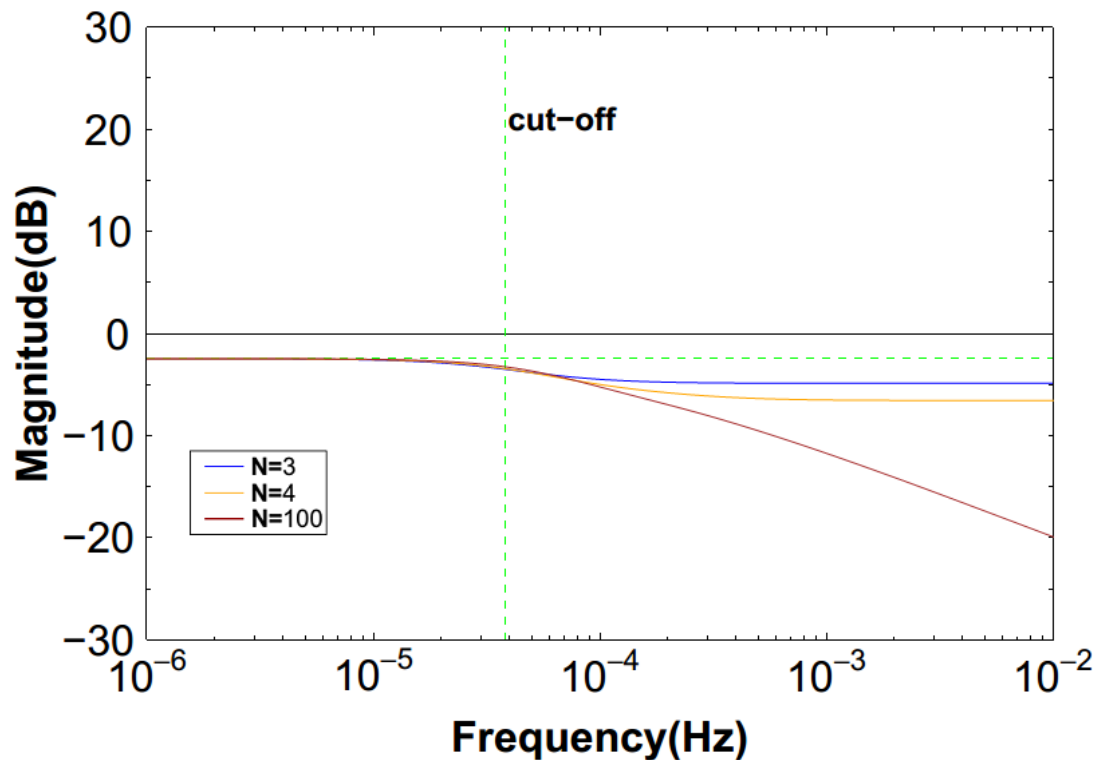
Input: Indoor temperature

Output: Inside heat flux density



Surface temperature as output

Input: Indoor temperature
Output: Inside surface temperature



CONCLUSIONS

- The **output** is **infinitely damped** for **strictly proper** transfer functions for **high frequencies**.
- The **output** can be **finitely amplified** for a **proper** transfer function for **high frequencies**.
- The use of **heat flux meters** is **limited**, **not** only for the measurement **device** **but** for the **heat flux** through the **wall** itself.
- The **outside heat flux density** will have a **major problem** to **high frequencies** present in **inputs** and/or **noise**.

Thanks for your attention

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